Name Key
Pre-AP Calculus
7.10 – Do Now

Date
Education is Freedom
Binder Section: DN

Do Now

1. Write the equation of a rational function $g(x)$ such that:

   a. $g(x)$ has a horizontal asymptote at $y = -4$.

   $g(x) = \frac{-4x^2 + \ldots}{x^2 + \ldots}$ Any function with equal leading degrees and a ratio of leading coefficients equal to -4

   b. $g(x)$ has a horizontal asymptote at $y = 0$.

   $g(x) = \frac{x + \ldots}{x^2 + \ldots}$ Any function with leading degree in denominator is greater than leading degree in numerator

   c. $g(x)$ does not have a horizontal asymptote.

   $g(x) = \frac{x^2 + \ldots}{x + \ldots}$ Any function with leading degree in denominator is less than leading degree in numerator

$$f(x) = \frac{5(x-2)(x+4)}{(x+2)(x+4)}$$

2. Given the function $f(x)$ shown above:

   a. Determine the equations of all asymptotes on the graph of $f(x)$.

   $y = 5, x = -2$

   b. Identify the coordinates of all y and x-intercepts of the graph of $f(x)$.

   $(0,-5); (2,0)$

   c. Determine the exact location of the hole on the graph of $f(x)$.

   $x = -4$ $\quad f(x) = \frac{5(x-2)\cancel{(x+4)}}{(x+2)\cancel{(x+4)}}$ $\quad \boxed{(-4, 15)}$

   $\lim_{x \to -4} \frac{5(x-2)}{x+2} = \frac{5(-4-2)}{-4+2} = \frac{5(-6)}{-2} = \frac{-30}{-2} = \boxed{+15}$

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Name ____________________ Seat # _______ Date _________________

Pre-AP Calculus Education is Freedom

7.10 – Mad Minute **Binder Section: MM**

Topic: Piecewise Function Analysis

Take #1

Goal Score: _____ /_______

Actual Score: _____ /_______

Met Goal? Yes / No

Goal for tomorrow: ___/___

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Mad Minute – Piecewise Function Analysis – Take #1

$$f(x) = \begin{cases} 4x+2 & x<0 \\ e^x+1 & 0<x\le 2 \\ x^2-3x & x>2 \end{cases}$$

Directions: Answer the following questions about the piecewise function given above.

1. $f(-4)$ $4(-4)+2$ $-16+2$ $\boxed{-14}$	2. $\lim\limits_{x\to 2^+} f(x)$ $(2)^2-3(2)$ $4-6$ $=\boxed{-2}$	3. $\lim\limits_{x\to 0} f(x)$ $4(0)+2 \overset{?}{=} e^0+1$ $2 = 1+1$ ✓ $\boxed{2}$
4. $f(0)$ $\boxed{\text{Undefined}}$	5. $f(2)$ $\boxed{e^2+1}$	6. $\lim\limits_{x\to 2} f(x)$ $e^2+1 \overset{?}{=} (2)^2-3(2)$ $e^2+1 = 4-6$ $e^2+1 \neq -2$ $\boxed{\text{Does Not Exist}}$

7. Which of the following correctly describes the graph of $f(x)$?

~~(A)~~ $f(x)$ is continuous at $x=0$ and has a removable discontinuity at $x=2$.
~~(B)~~ $f(x)$ is continuous at $x=0$ and has a jump discontinuity at $x=2$.
~~(C)~~ $f(x)$ has a jump discontinuity at $x=0$ and a removable discontinuity at $x=2$.
(D) $f(x)$ has a removable discontinuity at $x=0$ and a jump discontinuity at $x=2$.

Challenge! Find all the x-intercepts of $f(x)$.

$4x+2=0$
$4x=-2$
$x=-\frac{1}{2}$
$(-\frac{1}{2},0)$

$e^x+1=0$
$e^x=-1 \rightarrow x=\ln(-1)$
No Solution

$x^2-3x=0$
$x(x-3)=0$
$\cancel{x=0}$, $x=3$
not on interval/domain
$(3,0)$

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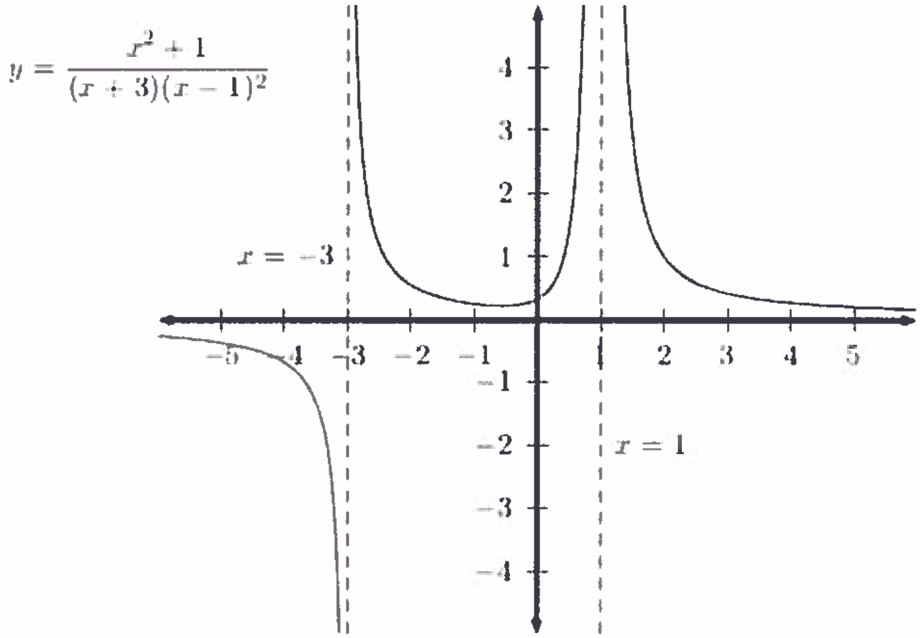
Pre-AP Calculus Education is Freedom
7.10 – Explore **Binder Section: EX**

Explore!



1. The graph of the function $y = \frac{x^2+1}{(x+3)(x-1)^2}$ is shown above. The graph has vertical asymptotes at $x = -3$ and $x = 1$, and a horizontal asymptote at $y = 0$, as shown. Based on the graph, find:

Problem	Solution	Problem	Solution
a. $\lim_{x\to -3^-} \frac{x^2+1}{(x+3)(x-1)^2}$	$-\infty$	d. $\lim_{x\to 1^-} \frac{x^2+1}{(x+3)(x-1)^2}$	$+\infty$
b. $\lim_{x\to -3^+} \frac{x^2+1}{(x+3)(x-1)^2}$	$+\infty$	e. $\lim_{x\to 1^+} \frac{x^2+1}{(x+3)(x-1)^2}$	$+\infty$
c. $\lim_{x\to -3} \frac{x^2+1}{(x+3)(x-1)^2}$	Does not exist	f. $\lim_{x\to 1} \frac{x^2+1}{(x+3)(x-1)^2}$	$+\infty$

2. Complete the following summarizer:

"The one-sided limit at a vertical asymptote must be either $+\infty$ or $-\infty$, whereas the two-sided limit at a vertical asymptote could be either $+\infty$, $-\infty$, or nonexistent."

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Pre-AP Calculus Education is Freedom
7.10 – Class Notes **Binder Section: CN**

Teacher Note

This is not a student facing notes page. Students should be using their Pre-AP Calculus notebook to capture their "I Do" and "We Do" Example

Example 1

$$f(x) = \frac{x^2 + 4x - 12}{x^3 + x^2}$$

1. Given the function $f(x)$ shown above, find each of the following:

a. $\lim\limits_{x \to -1^+} f(x) =$	d. $\lim\limits_{x \to 0^+} f(x) =$	g. $\lim\limits_{x \to +\infty} f(x) =$
b. $\lim\limits_{x \to -1^-} f(x) =$	e. $\lim\limits_{x \to 0^-} f(x) =$	h. $\lim\limits_{x \to -\infty} f(x) =$
c. $\lim\limits_{x \to -1} f(x) =$	f. $\lim\limits_{x \to 0} f(x) =$	

Example 2

$$g(x) = \frac{x^2 - 7x - 8}{(x^2 + 5x)(x^2 - 3x - 40)}.$$

2. Let $g(x)$ be the function shown above.
   a. Find all horizontal and vertical asymptotes of $g(x)$.
   b. Determine all one-sided and two-sided limits near each vertical asymptote on the graph of $g(x)$.

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Pre-AP Calculus — Education is Freedom

7.10 – Classwork — **Binder Section: CW**

Everybody Writes:

- The sign test can only be used for infinite limits to identify whether the one-sided limit is $+\infty$ or $-\infty$.
- $h(x)$ has a horizontal asymptote at $y = -3$ for $x \geq 0$. This must mean that $\lim_{x\to\infty} h(x) = -3$.

Problem Set A

1. Consider the function:

$$g(x) = \frac{4x^3 + 4x^2 - 8x}{2x^3 + 10x^2 - 28x} = \frac{4x(x^2+x-2)}{2x(x^2+5x-14)} = \frac{4x(x+2)(x-1)}{2x(x+7)(x-2)}$$

Evaluate each of the following limits, and show the work that leads to your answer.

Limit	Work	Answer
a. $\lim_{x\to -7^-} g(x)$	V.A. @ $x=-7$ $\frac{4(-7.1)(-7.1+2)(-7.1-1)}{2(-7.1)(-7.1+7)(-7.1-2)} = \frac{(-)(-)(-)}{(-)(-)(-)}$	$+\infty$
b. $\lim_{x\to -7^+} g(x)$	$\frac{4(-6.9)(-6.9+2)(-6.9-1)}{2(-6.9)(-6.9+7)(-6.9-2)} = \frac{(-)(-)(-)}{(-)(+)(-)}$	$-\infty$
c. $\lim_{x\to -7} g(x)$	$\lim_{x\to -7^-} g(x) \neq \lim_{x\to -7^+} g(x)$	Does not exist
d. $\lim_{x\to 2^+} g(x)$	V.A. @ $x=2$ $\frac{4(2.1)(2.1+2)(2.1-1)}{2(2.1)(2.1+7)(2.1-2)} = \frac{(+)(+)(+)}{(+)(+)(+)}$	$+\infty$
e. $\lim_{x\to 2^-} g(x)$	$\frac{4(1.9)(1.9+2)(1.9-1)}{2(1.9)(1.9+7)(1.9-2)} = \frac{(+)(+)(+)}{(+)(+)(-)}$	$-\infty$
f. $\lim_{x\to 2} g(x)$	$\lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$	Does Not Exist
g. $\lim_{x\to\infty} g(x)$	$g(x) \approx \frac{4x^3+\dots}{2x^3+\dots} = 2$	$\boxed{2}$
h. $\lim_{x\to -\infty} g(x)$	$g(x) \approx \frac{4x^3+\dots}{2x^3+\dots} = 2$	$\boxed{2}$

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Pre-AP Calculus — Education is Freedom

7.10 – Classwork — **Binder Section: CW**

Problem Set B

Directions: Solve each of the following problems. Show your work in the space below, and include a one-sentence justification in the box to the right.

1. $\lim_{n\to\infty} \frac{4n^2}{n^2 + 10{,}000n}$ is $\approx \frac{4n^2}{n^2} = 4$

   ~~(A) 0~~
   ~~(B) $\frac{1}{2{,}500}$~~
   ~~(C) 1~~
   (D) 4
   ~~(E) nonexistent~~

Justification

$f(x) = \frac{4x^2}{x^2 + 10{,}000x}$ has a horizontal asymptote at $y = 4$

2. Evaluate: $\lim_{x\to 4^-} \frac{x+4}{x-4}$

   $\frac{3.9+4}{3.9-4} = \frac{(+)}{(-)} = -\infty$

   (A) $-\infty$
   ~~(B) -1~~
   ~~(C) 0~~
   ~~(D) 1~~
   ~~(E) ∞~~

Justification

$f(x) = \frac{x+4}{x-4}$ has a vertical asymptote at $x = 4$, so one-sided limit is either $-\infty, +\infty$; sign test shows $-\infty$.

3. The continuous function h has domain $x > 0$ and range $y > 0$. If the asymptotes of the graph of h are $x = 6$ and $y = 3$, write one infinite limit and one limit at infinity involving $h(x)$.

   $\lim_{x\to\infty} h(x) = 3$

   $\lim_{x\to 6} h(x) = +\infty$

Justification

Horizontal asymptote at $y = 3$, w/ $x > 0$ domain, implies $\lim_{x\to+\infty} = 3$.
Vertical asymptote at $x = 6$, along w/ range $y > 0$, implies $\lim_{x\to 6} = +\infty$

4. Evaluate and show all work: $\lim_{x \to 1} \frac{2-x}{(x-1)^2}$.

~~(A)~~ $-\infty$

~~(B)~~ -1

~~(C)~~ 0

(D) ∞

~~(E)~~ Does not exist

$$\lim_{x \to 1^-} \frac{2-(0.9)}{(0.9-1)^2} \quad \frac{(+)}{(+)} = +\infty$$

$$\lim_{x \to 1^+} \frac{2-1.1}{(1.1-1)^2} \quad \frac{(+)}{(+)} = +\infty$$

Justification

Both one-sided limits at this vertical asymptote, $x=1$, are $+\infty$ according to sign tests.

5. Evaluate and show all work: $\lim_{x \to \infty} \frac{2-x}{(x-1)^2}$.

~~(A)~~ $-\infty$

~~(B)~~ -1

(C) 0

~~(D)~~ ∞

~~(E)~~ Does not exist

Justification

Leading degree in denominator > leading degree in numerator

6. Which of the following functions have a horizontal asymptote of $y = 0$? **Select all that apply.**

(A) $f(x) = \frac{1}{x}$

(B) $f(x) = \frac{10}{x^2+3}$

~~(C)~~ $f(x) = \frac{1+x}{6-x}$

(D) $f(x) = \frac{1+x}{6-x^2}$

~~(E)~~ $f(x) = \frac{(2x+1)(x-3)^2}{x^2+6x+5}$

(F) $f(x) = \frac{(2x+1)(x-3)}{x(x^2+6x+5)}$

Justification

These functions all have leading degrees in denominator > leading degrees in numerator.

7. Evaluate and show all work: $\lim_{x\to\infty} \frac{3x^2-x^3+7}{(2x+1)(x-1)^2}$

~~(A)~~ $-\infty$

(B) $-\frac{1}{2}$

~~(C)~~ 1

~~(D)~~ $\frac{3}{2}$

~~(E)~~ ∞

$$= \frac{3x^2-x^3+7}{2x^3+\dots}$$

$$\approx \frac{-x^3}{2x^3} = -1/2$$

Justification

$f(x) = \frac{3x^2-x^3+7}{(2x+1)(x-1)^2}$ has a horizontal asymptote at $y=-1/2$

8. Let $g(x) = \frac{5x^3-30x^2-80x}{-3x^3-15x^2}$.

$$g(x) = \frac{5x(x^2-6x-16)}{-3x^2(x+5)} = \frac{5x(x-8)(x+2)}{-3x^2(x+5)}$$

a. Find the equations of all asymptotes.

$y=-5/3,\ x=0,\ x=-5$

b. Evaluate $\lim_{x\to\infty} g(x)$. Justify your answer.

$\lim_{x\to\infty} g(x) = -5/3$ $g(x)$ has a horizontal asymptote at $y=-5/3$

c. Determine all one-sided and two-sided limits near each vertical asymptote. **Show all work and use correct limit notation.**

Two Vertical Asymptotes $\begin{cases} x=0 \\ x=-5 \end{cases}$

Work for $x=0$ shown below

$$\lim_{x\to -5^-} \frac{5(-5.1-8)(-5.1+2)}{-3(-5.1)(-5.1+5)} = \frac{(-)(-)}{(-)(-)(-)} = -\infty$$

$$\lim_{x\to -5^+} \frac{5(-4.9-8)(-4.9+2)}{-3(-4.9)(-4.9+5)} = \frac{(-)(-)}{(-)(-)(+)} = +\infty$$

$\lim_{x\to -5} g(x)$ does not exist

d. Evaluate $\lim_{x\to 0} g(x)$. Show the work that leads to your answer.

$$\lim_{x\to 0} \frac{5\cancel{x}(x-8)(x+2)}{-3x^{\cancel{2}}(x+5)} = \lim_{x\to 0} \frac{5(x-8)(x+2)}{-3x(x+5)}$$

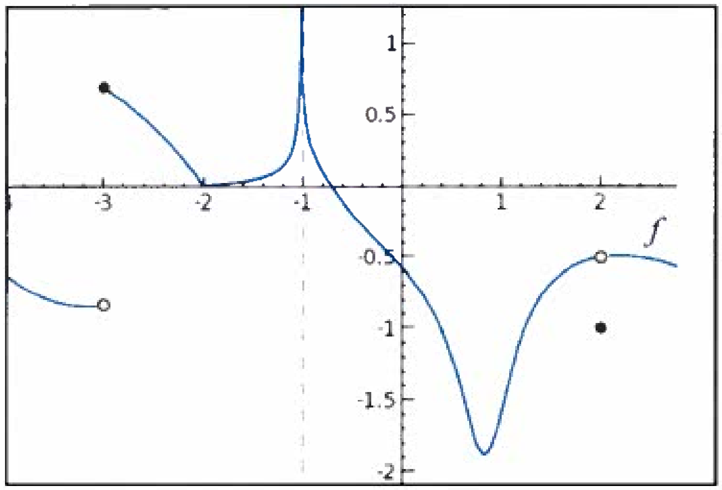
Vertical Asymptote at $x=0$

$$\lim_{x\to 0^-} \to \frac{5(-0.1-8)(-0.1+2)}{-3(-0.1)(-0.1+5)} = \frac{(-)(+)}{(-)(-)(+)} = -\infty$$

$$\lim_{x\to 0^+} \to \frac{5(0.1-8)(0.1+2)}{-3(0.1)(0.1+5)} = \frac{(-)(+)}{(-)(+)(+)} = +\infty$$

$\lim_{x\to 0} g(x)$ does not exist

Fast Workers! Nice Job working through Problem Set A and Problem Set B. Keep the math fresh by working through these spiral problems below.



1. The figure above shows the graph of a function $f(x)$, where f has a vertical asymptote at $x = -1$. Which of the following statements are true?

   I. $\lim_{x \to -3} f(x)$ exists ✗

   II. $\lim_{x \to 2} f(x)$ exists ✓

   III. $\lim_{x \to -1} f(x) = \infty$ ✓

   ~~(A)~~ II only

   ~~(B)~~ III only

   ~~(C)~~ I and II only

   (D) II and III only

2. The exact value of the expression $\sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$ is:

   ~~(A)~~ $-\frac{2\sqrt{3}}{3}$

   ~~(B)~~ $-\frac{\sqrt{3}}{2}$

   (C) $\frac{2\sqrt{3}}{3}$

   ~~(D)~~ $\frac{\sqrt{3}}{2}$

$$= \sec\left(-\pi/6\right)$$

$$= \frac{1}{\cos\left(-\pi/6\right)}$$

$$= \frac{1}{\sqrt{3}/2}$$

$$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

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Pre-AP Calculus Education is Freedom

7.10 – Exit Ticket **Binder Section: ET**

1. If $\lim_{x\to\infty} g(x) = -8$, then $g(x)$ has a horizontal asymptote at $y=-8$.

2. If $\lim_{x\to-2} h(x) = -\infty$, then $h(x)$ has a vertical asymptote at $x=-2$.

3. $\lim_{n\to\infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is $\approx \frac{3n^3}{n^3} = 3$

   ~~(A)~~ -5
   ~~(B)~~ -2
   ~~(C)~~ 1
   (D) 3
   ~~(E)~~ nonexistent

4. $\lim_{x\to-3^+} \frac{2-x}{x+3} =$ $\frac{2-(-2.9)}{-2.9+3} = \frac{(+)}{(+)} = +\infty$

   ~~(A)~~ $-\infty$
   ~~(B)~~ -1
   ~~(C)~~ 0
   ~~(D)~~ 1
   (E) ∞

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Pre-AP Calculus Education is Freedom

7.10 – Homework **Binder Section: HW**

Part I: New Material – Infinite Limits and Limits at Infinity

1. If $\lim_{x\to\infty} f(x) = 10$, then $f(x)$ has a horizontal asymptote at $y = 10$.

2. If $\lim_{x\to 5} f(x) = +\infty$, then $f(x)$ has a vertical asymptote at $x = 5$.

3. Consider the function $f(x) = \frac{3x^2-6x-45}{-x^2-5x+6}$. $= \frac{3(x^2-2x-15)}{-(x^2+5x-6)} = \frac{3(x-5)(x+3)}{-(x+6)(x-1)}$

   a. Evaluate: $\lim_{x\to\infty} f(x) = -3$

   b. Find the horizontal asymptote of $f(x)$. Explain why this is correct.

   $y = -3$; the leading degrees in the numerator and denominator are equal, so the horizontal asymptote is formed by the ratio of the leading coefficients

   c. Find all vertical asymptotes of $f(x)$. Explain why these are correct.

   $x = -6$, $x = 1$ These are factors that appear only in the denominator.

   d. Evaluate all one-sided and two-sided limits near all vertical asymptotes. **Show all work neatly.**

$x = -6$: $\lim_{x\to -6^-} f(x) = \frac{3(-6.1-5)(-6.1+3)}{-(-6.1+6)(-6.1-1)} = \frac{(-)(-)}{(-)(-)(-)} = -\infty$

$\lim_{x\to -6^+} f(x) = \frac{3(-5.9-5)(-5.9+3)}{-(-5.9+6)(-5.9-1)} = \frac{(-)(-)}{(-)(+)(-)} = +\infty$

$\rightarrow \lim_{x\to -6} f(x)$ does not exist

$x = 1$: $\lim_{x\to 1^-} f(x) = \frac{3(0.9-5)(0.9+3)}{-(0.9+6)(0.9-1)} = \frac{(-)(+)}{(-)(+)(-)} = -\infty$

$\lim_{x\to 1^+} f(x) = \frac{3(1.1-5)(1.1+3)}{-(1.1+6)(1.1-1)} = \frac{(-)(+)}{(-)(+)(+)} = +\infty$

$\rightarrow \lim_{x\to 1} f(x)$ does not exist

4. Consider the function $g(x) = \frac{2-x}{(x-5)^3}$

a. State the horizontal asymptote of $g(x)$. Show how you got your answer.

$y = 0$ → leading degree in denominator, 3, is greater than leading degree in numerator, 1.

b. Evaluate each of the following and **show all steps**:

a) $\lim_{x\to5^+} g(x)$ $\quad \frac{2-(5.1)}{(5.1-5)^3} = \frac{(-)}{(+)} = -\infty$

b) $\lim_{x\to5^-} g(x)$ $\quad \frac{2-4.9}{(4.9-5)^3} = \frac{(-)}{(-)} = +\infty$

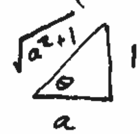
c) $\lim_{x\to5} g(x)$

Does not exist

d) $\lim_{x\to\infty} g(x) = 0$.

Part II: Spiral Material – keep the math fresh!

5. Evaluate each of the following expressions without a calculator:

a) $\csc\left(-\frac{11\pi}{6}\right) = -\frac{1}{\sin\frac{11\pi}{6}}$ $= +2$	b) $\tan\left(\frac{11\pi}{4}\right) = \tan\left(\frac{3\pi}{4}\right)$ $= -1$	c) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ $= \frac{3\pi}{4}$
d) $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$ Undefined	e) $\sec\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ $= \sec\left(-\frac{\pi}{3}\right)$ $= \frac{1}{\cos\pi/3} = +2$	f) $\sin\left(\arctan\left(\frac{1}{a}\right)\right) = \sin\theta$  $= \boxed{\frac{1}{\sqrt{a^2+1}}}$

7.10 - Infinite Limits and Limits at Infinity [Date]

Infinite Limits

A function $f(x)$ is said to have a vertical asymptote at $x=a$ if and only if $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$.

This limit taken at a vertical asymptote is known as an infinite limit.

Limits at Infinity

A function $f(x)$ is said to have a horizontal asymptote at $y=L$ if and only if $\lim_{x \to -\infty} f(x) = L$ or $\lim_{x \to \infty} f(x) = L$.

This limit taken as x approaches $\pm\infty$ is known as a limit at infinity, as well as an end behavior limit.

Example 1

$$f(x) = \frac{x^2+4x-12}{x^3+x^2}$$

Steps for Infinite Limits

1. Factor Completely, Cancel Holes
2. Choose x-value close to limit value on left/right, according to limit
3. Evaluate, but focus only on sign, since the answer will be either $-\infty$ or $+\infty$

a) $\lim_{x \to -1^+} f(x)$

$= \lim_{x \to -1^+} \frac{x^2+4x-12}{x^3+x^2}$

$= \lim_{x \to -1^+} \frac{(x+6)(x-2)}{x^2(x+1)}$

Right of -1 → choose $x = -0.9$

Sign Test: $\approx \frac{(-0.9+6)(-0.9-2)}{(-0.9)^2(-0.9+1)} = \frac{(+)(-)}{(+)(+)} = \boxed{-\infty}$

b) $\lim_{x \to -1^-} f(x)$

Left of -1 → choose $x = -1.1$

$\approx \frac{(-1.1+6)(-1.1-2)}{(-1.1)^2(-1.1+1)} = \frac{(+)(-)}{(+)(-)} = \boxed{+\infty}$

(cont.) →

c) $\lim_{x \to -1} f(x)$ = does not exist since the left-hand limit does not equal the right-hand limit, $-\infty \neq +\infty$.

7.10 (cont.) $f(x) = \dfrac{x^2+4x-12}{x^3+x^2} = \dfrac{(x+6)(x-2)}{x^2(x+1)}$

d) $\lim\limits_{x \to 0^+} f(x)$ $x = 0$ is a vertical asymptote

Choose $x = 0.1 \rightarrow \dfrac{(0.1+6)(0.1-2)}{(0.1)^2(0.1+1)} = \dfrac{(+)(-)}{(+)(+)} = -\infty$

e) $\lim\limits_{x \to 0^-} f(x)$ Choose $x = -0.1 \rightarrow \dfrac{(-0.1+6)(-0.1-2)}{(-0.1)^2(-0.1+1)} = \dfrac{(+)(-)}{(+)(+)} = -\infty$

f) $\lim\limits_{x \to 0} f(x) = \boxed{-\infty}$ since both $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = -\infty$.

g) $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{x^2+4x-12}{x^3+x^2} \approx \dfrac{x^2+\ldots}{x^3+\ldots} = \lim\limits_{x \to \infty} \dfrac{x^2}{x^3} = \dfrac{1}{x} = 0.$

isolate the leading terms; these are the only terms that will matter as $x \to \infty$.

h) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \dfrac{x^2}{x^3} = 0.$

$\rightarrow$ $y = 0$ is a horizontal asymptote, so the end behavior of this function will tend towards this asymptote.

<u>Example 2</u>

$$g(x) = \frac{x^2-7x-8}{(x^2+5x)(x^2-3x-40)}$$

2. Let $g(x)$ be the function shown above. Find all horizontal and vertical asymptotes of $g(x)$.

$g(x) = \dfrac{\cancel{(x-8)}(x+1)}{x(x+5)\cancel{(x-8)}(x+5)}$ Vertical Asymptotes: $x = -5, x = 0$
Horizontal Asymptote: $y = 0$

b. Determine one-sided and two-sided limits near each vertical asymptote.

$x = 0$

$\lim\limits_{x \to 0^-} g(x) = \dfrac{(-0.1+1)}{(-0.1)(-0.1+5)^2} = \dfrac{+}{(-)(+)} = -\infty$

$\lim\limits_{x \to 0^+} g(x) = \dfrac{(+0.1+1)}{(+0.1)(0.1+5)^2} = \dfrac{+}{(+)(+)} = +\infty$

$\lim\limits_{x \to 0} g(x)$ does not exist

$x = -5$

$\lim\limits_{x \to -5^-} g(x) = \dfrac{(-5.1+1)}{(-5.1)(-5.1+5)^2} = \dfrac{(-)}{(-)(+)} = +\infty$

$\lim\limits_{x \to -5^+} g(x) = \dfrac{(-4.9+1)}{(-4.9)(-4.9+5)^2} = \dfrac{(-)}{(-)(+)} = +\infty$

$\lim\limits_{x \to -5} g(x) = +\infty$