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Name Key
Pre-AP Calculus
11.04 – DN + Explore

Date
Education is Freedom
Binder Section: EX

Do Now + Explore!

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	−120	−150

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

a) Use the data in the table to estimate $v_A'(10)$.

$$v_A'(10) = \frac{v_A(12) - v_A(8)}{12-8} = \frac{-150+120}{4} = \frac{-30}{4} = -7.5$$

b) Interpret the meaning of $v_A'(10)$ in the context of the problem.

$v_A'(10)$ represents the instantaneous rate of change of velocity, in meters per minute per minute, of the Train A ~~at~~ after 10 minutes.

c) Selected values of Train A's velocity are given in meters per minute.

a. What units would be used to describe the displacement of the train?

Meters

b. What units would be used to describe the derivative of the velocity of the train?

Meters per minute per minute
or meters per minute squared m/min^2

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Pre-AP Calculus Education is Freedom

11.04 – Mad Minute **Binder Section: MM**

Topic: Sign Charts

Take #4

Goal Score: ______ /______

Actual Score: ______ /______

Met Goal? Yes / No

Goal for tomorrow: ___/___

Mad Minute – Sign Charts – Take #4

Directions: Use the two sign charts below to answer the questions that follow about $f(x)$.

$f'(x)$								
0	$-$	$\frac{1}{e^2}$	$+$	e	$+$	e^3	$-$	$+\infty$

$f''(x)$						
0	$-$	$\frac{1}{e}$	$-$	e^2	$+$	$+\infty$

1. At what value(s) of x does f have a relative minimum? $x = 1/e^2$	2. On what interval(s) is f concave up? $(e^2, +\infty)$	3. On what interval(s) is f increasing? $(1/e^2, e^3)$
4. At what value(s) of x does f have a point of inflection? $x = e^2$	5. On what interval(s) is f concave down? $(0, e^2)$	6. At what value(s) of x does f have a relative maximum? $x = e^3$
7. At what value(s) of x does f have a critical point that is not an extrema? $x = e$	8. On what interval(s) is f both concave down and increasing? $(1/e^2, e^2)$	9. On what interval(s) is f decreasing with an increasing slope? $(e^3, +\infty)$

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Pre-AP Calculus Education is Freedom

11.04 –Class Notes **Binder Section: CN**

Teacher Note

This is not a student facing notes page. Students should be using their Pre-AP Calculus notebook to capture their "I Do" and "We Do" Example

Example 1

1. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = -t^3 + 7t^2 - 14t + 8$, where s is measured in meters and t is measured in seconds.

   a) Find the displacement of the particle during the first 2 seconds.

   b) Find the average velocity of the particle during the first 4 seconds.

   c) Find the instantaneous velocity of the particle when $t = 4$.

   d) Find the acceleration of the particle when $t = 4$.

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Pre-AP Calculus Education is Freedom

11.04 – Classwork **Binder Section: CW**

Problem Set A

1. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in seconds.

a) Find the instantaneous velocity of the particle when $t = 4$.

$s'(t) = v(t) = 2t - 4$

$v(4) = 2(4) - 4 = \boxed{4\ m/s}$

b) Find the acceleration of the particle when $t = 4$.

$s''(t) = v'(t) = a(t) = 2$

$a(4) = \boxed{2\ m/s^2}$

c) Find the displacement of the particle during the first 2 seconds.

$s(2) = 2^2 - 4(2) + 3$

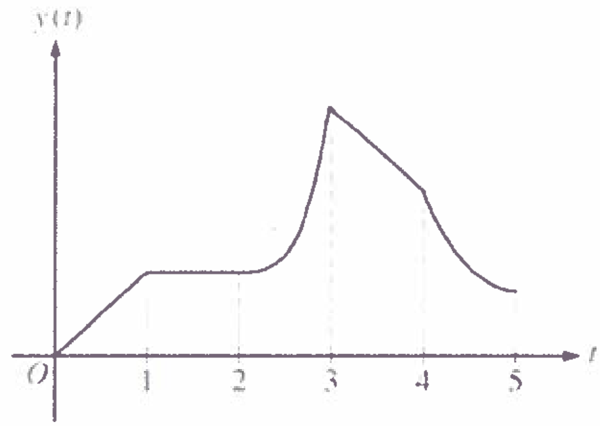
$= 4 - 8 + 3$

$= \boxed{-1\ m}$

d) Find the average velocity of the particle during the first 4 seconds.

$\frac{s(4) - s(0)}{4 - 0} = \frac{(4^2 - 4(4) + 3) - (0^2 - 4(0) + 3)}{4} = \frac{16 - 16 + 3 - 3}{4} = \boxed{0\ m/s}$

Example 2 – AP Multiple Choice



2. A particle moves along the y-axis. The graph of the particle's position $y(t)$ at time t is shown above for $0 \le t \le 5$. For what values of t is the velocity of the particle negative and the acceleration positive?

$v(t) = y'(t) < 0 \rightarrow y(t)$ is decreasing
$a(t) = y''(t) > 0 \rightarrow y(t)$ is concave up

~~(A)~~ $0 < t < 1$
~~(B)~~ $1 < t < 2$
~~(C)~~ $2 < t < 3$
~~(D)~~ $3 < t < 4$
(E) $4 < t < 5$

Justify your answer.

On $4<t<5$ $y(t)$ is both decreasing and concave up which satisfies $y'(t)=v(t)$ is negative, when $y'(t)$ is decreasing, and $y''(t) = a(t)$ is positive, when $y(t)$ is concave up.

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Pre-AP Calculus

11.04 – Classwork

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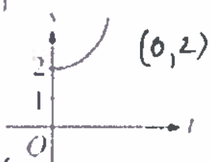
Binder Section: CW

Problem Set B

1. A particle starts from rest at the point $(2, 0)$ and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t? **Justify your answer.**

~~(A)~~

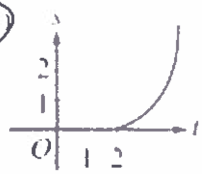


(0,2)

- positive acceleration = $s''(t) > 0$
  - $\rightarrow$ $s(t)$ is concave up
- $s(t)$ goes through $(2,0)$

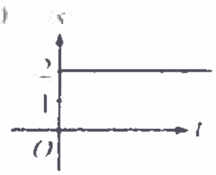
~~(B)~~

(C)



~~(D)~~

~~(E)~~



2. A particle moves along a straight line so that at time $t > 0$ the position of the particle is given by $s(t)$, the velocity is given by $v(t)$, and the acceleration is given by $a(t)$. Which of the following expressions gives the average velocity of the particle on the interval $[2, 8]$?

$$\frac{s(8)-s(2)}{8-2} = \frac{s(8)-s(2)}{6}$$

(A) $\frac{s(8)-s(2)}{6}$ ✓

~~(B)~~ $\frac{v(8)-v(2)}{6}$ average acceleration

~~(C)~~ $\frac{a(8)-a(2)}{6}$ average jerk

~~(D)~~ $v(8) - v(2)$ change in velocity

~~(E)~~ $a(8) - a(2)$ change in acceleration

t (hours)	0	1	2	3	4	5	6
$s(t)$ (miles)	0	25	55	92	150	210	275

2. The table above gives the distance $s(t)$, in miles, that a car has traveled at various times t, in hours, during a 6-hour trip. The graph of the function s is increasing and concave up. Based on the information, which of the following could be the velocity of the car, in miles per hour, at time $t = 3$?

$s'(t) = v(t)$

$v(2.5) \approx \frac{s(3)-s(2)}{3-2} < v(3) < v(3.5) \approx \frac{s(4)-s(3)}{4-3}$

$\frac{92-55}{3-2} < v(3) < \frac{150-92}{4-3}$

$37 < v(3) < 58$

~~(A)~~ 37
(B) 49
~~(C)~~ 58
~~(D)~~ 65
~~(E)~~ 92

Justify your answer.

Since $v(t)$ represents $s'(t)$, and s is increasing and concave up, $v(t)$ is increasing, so $v(3)$ must be greater than $v(2.5)$ and less than $v(3.5)$.

3. A particle moves along the x-axis such that its position $x(t)$ is given by $x(t) = \frac{1}{5}t^5 - \frac{1}{2}t^4 - 2t^2$. Find the particle's velocity at $t = 1$ and the particle's acceleration at $t = 1$. Is the speed of the particle increasing, decreasing, or neither? Explain your reasoning.

$x(t) = \frac{1}{5}t^5 - \frac{1}{2}t^4 - 2t^2$

$x'(t) = v(t) = t^4 - 2t^3 - 4t$

$v(1) = 1^4 - 2(1)^3 - 4(1)$

$= 1 - 2 - 4 = \boxed{-5}$

$a(t) = x''(t) = v'(t) = 4t^3 - 6t^2 - 4$

$a(1) = 4(1)^3 - 6(1)^2 - 4$

$= 4 - 6 - 4 = \boxed{-6}$

The speed of the particle is increasing because the velocity and acceleration have the same signs, both $v(1)$ and $a(1) < 0$.

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Fast Workers! Nice Job working through Problem Set A and Problem Set B. Keep the math fresh by working through these spiral problems below.

Composite Functions Review

1. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x)) =$

~~1.~~ $5x^2 + 15x + 25$
~~2.~~ $5x^3 + 15x^2 + 20x + 25$
~~3.~~ 1125
~~4.~~ 225
E. 5

2. If $f(g(x)) = \ln(x^2 + 4)$, $f(x) = \ln(x^2)$, and $g(x) > 0$ for all real x, then $g(x) =$

~~(A)~~ $\frac{1}{\sqrt{x^2+4}}$
~~(B)~~ $\frac{1}{x^2+4}$
(C) $\sqrt{x^2+4}$
~~(D)~~ $x^2 + 4$
~~(E)~~ $x + 2$

$f(x) = \ln(x^2)$
$f(g(x)) = \ln(x^2+4)$
$g(x) = (x^2+4)^{1/2} = \sqrt{x^2+4}$

3. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

~~(A)~~ $3x^3 - |x|$
~~(B)~~ $|3x^2 - 1|$
~~(C)~~ $3x^2|x| - 1$
~~(D)~~ $3|x| - 1$
(E) $3x^2 - 1$

$h(x) = 3(|x|)^2 - 1$
$= 3x^2 - 1$

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Pre-AP Calculus — Education is Freedom

11.04 – Exit Ticket — **Binder Section: ET**

1. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = 2t^2 - 4\cos t$, where s is measured in miles and t is measured in hours.

   a) Find the instantaneous velocity of the particle when $t = \frac{\pi}{4}$.

   $v(t) = s'(t) = 4t + 4\sin t$

   $v(\pi/4) = 4(\pi/4) + 4\sin(\pi/4)$

   $= \pi + 4 \cdot \sqrt{2}/2 = \boxed{\pi + 2\sqrt{2} \text{ miles/hour}}$

   b) Find the acceleration of the particle when $t = \frac{2\pi}{3}$.

   $v'(t) = s''(t) = a(t) = 4 + 4\cos t$

   $a(2\pi/3) = 4 + 4\cos(2\pi/3)$

   $= 4 + 4(-1/2) = 4 - 2 = \boxed{2 \text{ miles/hours}^2}$

2. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average [redacted] of the particle from $t = 0$ to $t = 8$?

   acceleration

   $$\frac{v(8) - v(0)}{8 - 0}$$

   ~~(A)~~ $\frac{a(8)-a(0)}{8}$

   ~~(B)~~ $\frac{a(8)-a(0)}{8}$

   ~~(C)~~ $\frac{v(0)+v(8)}{2}$

   (D) $\frac{v(8)-v(0)}{8}$

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Pre-AP Calculus

Education is Freedom

11.04 – Homework

Binder Section: HW

Part I: New Material – Particle Motion Revisited

1. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \leq t \leq 3$ is

$s(t) = -5t^2$

$\frac{s(3)-s(0)}{3-0} = \frac{-5(3)^2+5(0)^2}{3}$

$= \frac{-5\cdot 9}{3} = -15$

~~(A)~~ -45
~~(B)~~ -30
(C) -15
~~(D)~~ -10
~~(E)~~ -5

2. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = -2t^3 + 3t^2 - 5t$, where s is measured in meters and t is measured in seconds.

a) Find the displacement of the particle during the first 3 seconds.

$s(3) = -2(3)^3 + 3(3)^2 - 5(3)$
$= -2(27) + 27 - 15 = -54 + 27 - 15 = \boxed{-42 \text{ meters}}$

b) Find the average velocity of the particle during the first 3 seconds.

$\frac{s(3)-s(0)}{3-0} = \frac{-42-0}{3} = \boxed{-14 \text{ m/s}}$

c) Find the instantaneous velocity of the particle when $t = 3$.

$s'(t) = v(t) = -6t^2 + 6t - 5$
$v(3) = -6(3^2) + 6(3) - 5$
$= -54 + 18 - 5 = \boxed{-41 \text{ m/s}}$

d) Find the acceleration of the particle when $t = 3$.

$s''(t) = a(t) = -12t + 6$
$a(3) = -12(3) + 6$
$= -36 + 6 = \boxed{-30 \text{ m/s}^2}$

Part II: Spiral Material – keep the math fresh!

3. Let f be a differentiable function whose derivative is given by the equation $f'(x) = x^2e^x - xe^x$. Which of the following statements about f is true?

$f'(x) = 0 = xe^x(x-1)$

$f'(x)$: $+ \quad - \quad +$ on $-\infty \quad 0 \quad 1 \quad +\infty$

~~(A)~~ f has inflection points at $x = 0$ and $x = -1$.
~~(B)~~ f has inflection points at $x = 0$ and $x = 1$.
(C) f has a relative maximum at $x = 0$ and relative minimum at $x = 1$.
~~(D)~~ f has a relative minimum at $x = 0$ and a relative maximum at $x = 1$.
~~(E)~~ f has a relative minimum at $x = -1$ and a relative maximum at $x = 1$.

4. The average rate of change of $f(x) = x^3$ over the interval $[a, b]$ is

$$\frac{f(b)-f(a)}{b-a} = \frac{b^3-a^3}{b-a} = b^2 + ab + a^2$$

~~(A)~~ $3b + 3a$
(B) $b^2 + ab + a^2$
~~(C)~~ $\frac{b^2+a^2}{2}$
~~(D)~~ $\frac{b^3-a^3}{2}$
~~(E)~~ $\frac{b^4-a^4}{4(b-a)}$

$y = x^2/2$

5. The point on the curve $2y = x^2$ nearest to $(4, 1)$ is

~~(A)~~ $(0, 0)$
(B) $(2, 2)$
~~(C)~~ $(\sqrt{2}, 1)$
~~(D)~~ $(2\sqrt{2}, 4)$
~~(E)~~ $(4, 8)$

$$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$D = \sqrt{(x-4)^2 + (x^2/2 - 1)^2}$$

$$D' = \frac{1}{2\sqrt{(x-4)^2 + (x^2/2-1)^2}} \cdot (2)(x-4)' + 2(x^2/2 - 1)' \cdot 2x/2$$

$$0 = \frac{2(x-4) + 2x(x^2/2 - 1)}{2\sqrt{(x-4)^2 + (x^2/2-1)^2}}$$

$$0 = 2(x-4) + 2x(x^2/2 - 1)$$

$$0 = 2x - 8 + x^3 - 2x$$

$$0 = -8 + x^3$$

$$8 = x^3$$

$$\boxed{x = 2}$$

$$2y = x^2 \rightarrow 2y = (2)^2$$

$$2y = 4$$

$$\boxed{y = 2}$$

11.04 - Particle Motion Day 1 [Date]

New Definition - Particle Motion Relationships

- If $s(t)$ represents the displacement or position of a particle moving along a line at any time $t \geq 0$, with s measured in meters and t measured in seconds, then:
  - $s'(t) = v(t)$ which represents the velocity of the particle, measured in meters per second
  - $s''(t) = v'(t) = a(t)$ which represents the acceleration of the particle measured in meters per second squared.

[Note - discuss key ideas here aloud]

Example 1

1. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = -t^3 + 7t^2 - 14t + 8$, where s is measured in meters ~~at~~ and t is measured in seconds.

a) Find the displacement of the particle after the first 2 seconds.

$$s(2) = -(2)^3 + 7(2)^2 - 14(2) + 8$$
$$= -8 + 28 - 28 + 8 = 0 \quad \boxed{0 \text{ meters}}$$

b) Find the average velocity of the particle during the first four seconds.

Average rate of change of position = average velocity $= \dfrac{s(4) - s(0)}{4 - 0}$

$$s(4) = -(4)^3 + 7(4)^2 - 14(4) + 8$$
$$= -64 + 112 - 56 + 8 = 0$$
$$s(0) = 8$$

$$= \frac{0 - 8}{4 - 0} = \frac{-8}{4} = -2 \quad \boxed{-2 \text{ meters per second}}$$

c) Find the velocity of the particle when $t = 4$.

$$v(t) = s'(t) = -3t^2 + 14t - 14$$
$$v(4) = -3(4)^2 + 14(4) - 14 = -48 + 56 - 14 = \boxed{-6 \text{ meters per second}}$$

d) Find the acceleration of the particle when $t = 4$.

$$a(4) = v'(4) = s''(4)$$
$$a(t) = v'(t) = -6t + 14$$
$$a(4) = -6(4) + 14$$
$$= -24 + 14 = \boxed{-10 \text{ meters per second squared}}$$