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Name Key
Pre-AP Calculus
10.07 – Do Now

Date
Education is Freedom
Binder Section: DN

Do Now

1. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7
(B) ~~$-\frac{3}{4}$~~
(C) ~~0~~
(D) ~~2~~
(E) ~~6~~

$g'(x) = 12x^2 + 6x - 6$
$0 = 6(2x^2 + x - 1)$
$0 = 6(2x-1)(x+1)$
$x = 1/2,\ x = -1$

Candidates: $x = -2, x = -1, x = 1/2, x = 1$

x	f(x)
-2	$4(-2)^3 + 3(-2)^2 - 6(-2) + 1 = -7$ Min
-1	$4(-1)^3 + 3(-1)^2 - 6(-1) + 1 = 6$ Max
1/2	$4(1/2)^3 + 3(1/2)^2 - 6(1/2) + 1 = \frac{1}{2} + \frac{3}{4} - 3 + 1 = -0.75$
1	$4(1)^3 + 3(1)^2 - 6(1) + 1 = 2$

2. Find the locations of the absolute extrema of $h(x) = \frac{x^2}{x-1}$ on the closed interval $\left[-1, \frac{1}{2}\right]$.

$h'(x) = \frac{(x-1)2x - x^2(1)}{(x-1)^2}$

$h'(x) = \frac{2x^2 - 2x - x^2}{(x-1)^2}$

$0 = \frac{x^2 - 2x}{(x-1)^2}$

$0 = x^2 - 2x$
$0 = x(x-2)$
$x = 0$
~~$x = 2$~~ not in interval

Candidates: $x = 0, x = -1, x = 1/2$

x	f(x)
-1	$(-1)^2/(-1-1) = -1/2$ Min
0	$0^2/(0-1) = 0$ Max
1/2	$(1/2)^2/(1/2 - 1) = \frac{1/4}{-1/2} = -1/2$ Min

Absolute Minimum of $-1/2$ at $x = -1$ and $x = 1/2$
Absolute Maximum of 0 at $x = 0$

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Name ____________________ Seat # _______ Date ________________

Pre-AP Calculus Education is Freedom

10.07 – Mad Minute **Binder Section: MM**

Topic: Graphic Analysis
Translations

Take #2

Goal Score: _____ /______

Actual Score: _____ /______

Met Goal? Yes / No

Goal for tomorrow: ___/___

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Mad Minute – Graphic Analysis Translations – Take #2

Directions: Fill-in-the-blanks with the correct graphic analysis term.

1. If $f''(x) < 0$, then $f(x)$ is concave down	2. If $f'(x) < 0$, then $f(x)$ is decreasing	3. If $f''(x) > 0$, then $f'(x)$ is increasing	4. If $f''(c) = 0$, then $x = c$ could be a inflection point
5. If $f(x)$ is concave up, then $f''(x)$ is positive	6. If $f(x)$ is increasing, then $f'(x)$ is positive	7. If $f'(c) = 0$ and $f''(c) < 0$, then $x = c$ is a relative maximum	8. If $f''(x) > 0$, then $f(x)$ is concave up
9. If $f'(x)$ is decreasing, then $f''(x)$ is negative	10. If $f(x)$ is concave down, then $f'(x)$ is decreasing	11. If $f'(x)$ is increasing, then $f(x)$ is concave up	12. If $f'(x) > 0$, then $f(x)$ is increasing
13. If $f'(c) = 0$, then $x = c$ is a horizontal tangent	14. If $f(x)$ is decreasing, then $f'(x)$ is negative	15. If $f'(c) = 0$ and $f''(c) > 0$, then $x = c$ is a relative minimum	16. If $f'(x)$ is increasing, then $f''(x)$ is positive

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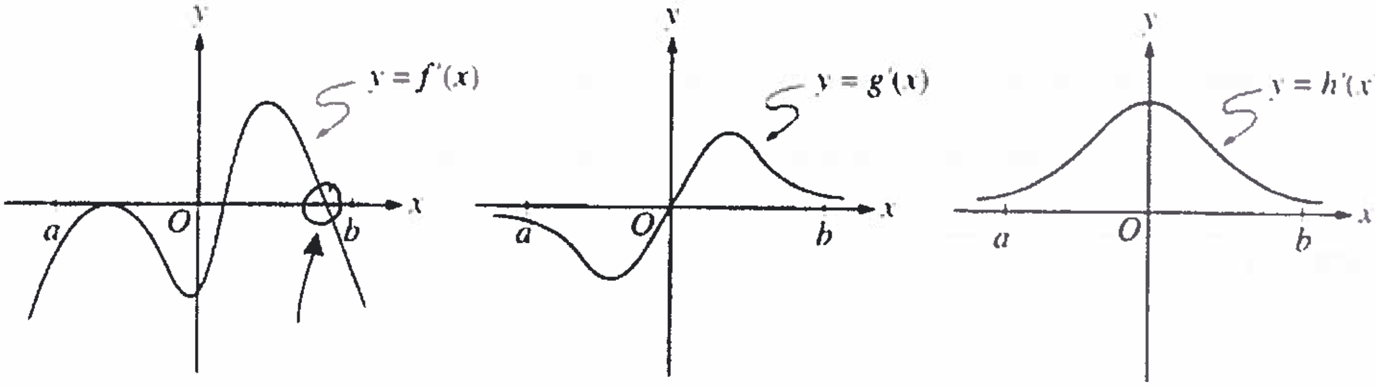
Name ____________________
Pre-AP Calculus
10.07 – Explore

Date ________________
Education is Freedom
Binder Section: EX

Explore!



1. The graphs of the derivatives of the functions, f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

   (A) f only
   ~~(B)~~ g only
   ~~(C)~~ h only
   ~~(D)~~ f and g only
   ~~(E)~~ $f, g,$ and h

   Explain your reasoning.

Only f has a relative maximum, because only f' changes from positive to negative

g'(x) changes from negative to positive but not positive to negative, and h'(x) stays positive on (a, b).

Name ____________________ Seat # _______ Date ________________

Pre-AP Calculus

Education is Freedom

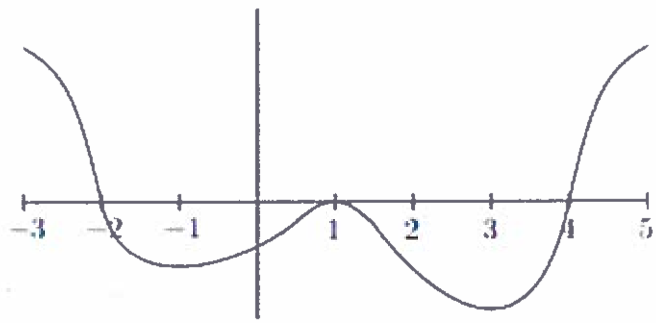
10.07 –Class Notes

Binder Section: CN

Teacher Note

This is not a student facing notes page. Students should be using their Pre-AP Calculus notebook to capture their "I Do" and "We Do" Example

Example 1



1. The figure above shows the graph of f', the derivative of a function f. Identify:

   a) The x-values at which f has a relative maximum $x = -2$

   b) The intervals on which f is concave down. $(-3,-1)$ and $(1,3)$

   c) The x-values at which the graph of f has a point of inflection. $x = -1$, $x = 1$, and $x = 3$

   d) The intervals on which f is increasing. $(-3,-2)$ and $(4,5)$

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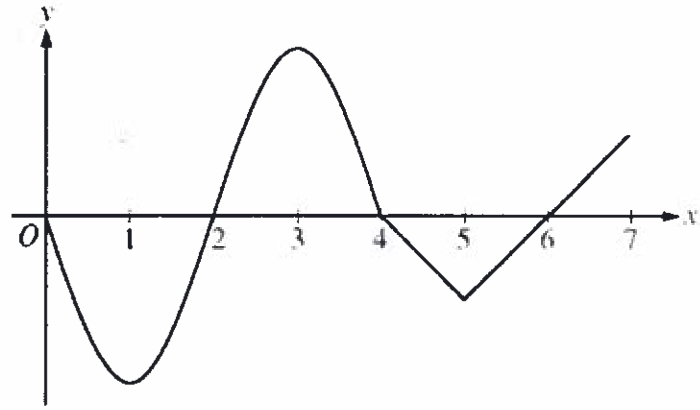
Name ____________________ Seat # ______ Date ________________

Pre-AP Calculus — Education is Freedom

10.07 – Classwork — **Binder Section: CW**

Problem Set A

1. The graph of f', the derivative of the function f, is shown to the right. On which of the following intervals is f decreasing?

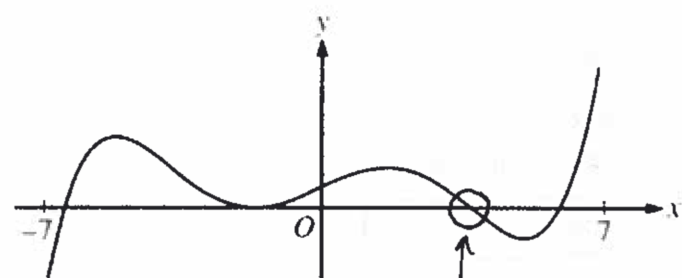


Graph of f'

~~(A)~~ $[2,4]$ only
~~(B)~~ $[3,5]$ only
~~(C)~~ $[0,1]$ and $[3,5]$
~~(D)~~ $[2,4]$ and $[6,7]$
(E) $[0,2]$ and $[4,6]$

Justify your answer: f is decreasing on $[0,2]$ and $[4,6]$ because f' is negative on $[0,2]$ and $[4,6]$.

2. The figure to the right shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?



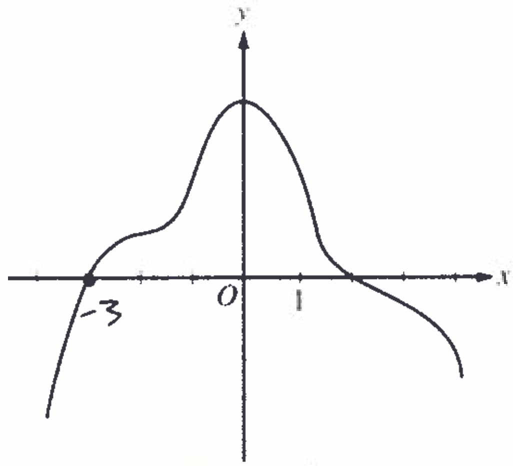
Graph of f'

relative maximum

(A) One
~~(B)~~ Two
~~(C)~~ Three
~~(D)~~ Four
~~(E)~~ Five

Justify your answer: f has one relative maximum, at around $x = 3.5$, because this is the only instance at which f' changes from positive to negative.

Example 2 – AP Multiple Choice!



Graph of f'

1. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

   I. f has a relative minimum at $x = -3$. ✓ f' changes from negative to positive

   II. The graph of f has a point of inflection at $x = -2$. ✗ f' does not have an extrema

   III. The graph of f is concave down for $0 < x < 4$. ✓ f' is decreasing

~~(A)~~ I only

~~(B)~~ II only

~~(C)~~ III only

~~(D)~~ I and II only

(E) I and III only

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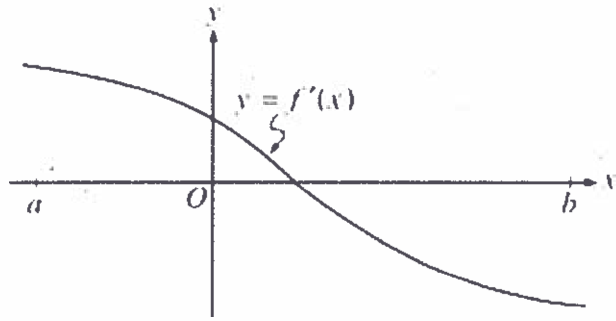
Name ____________________ Seat # _______ Date _________________

Pre-AP Calculus Education is Freedom

10.07 – Classwork **Binder Section: CW**

Problem Set B



1. The graph of f', the derivative of the function f, is shown in the figure above. Which of the following statements must be true?

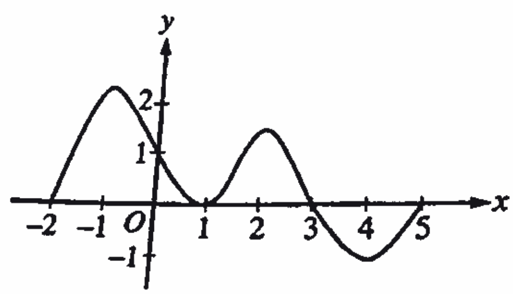
   ✓ I. f is continuous on the open interval (a, b). ✓ f' is continuous on (a, b)

   ~~II.~~ f is decreasing on the open interval (a, b). × f' is positive, then negative

   ✓ III. The graph of f is concave down on the open interval (a, b). ✓ f' is decreasing on (a, b)

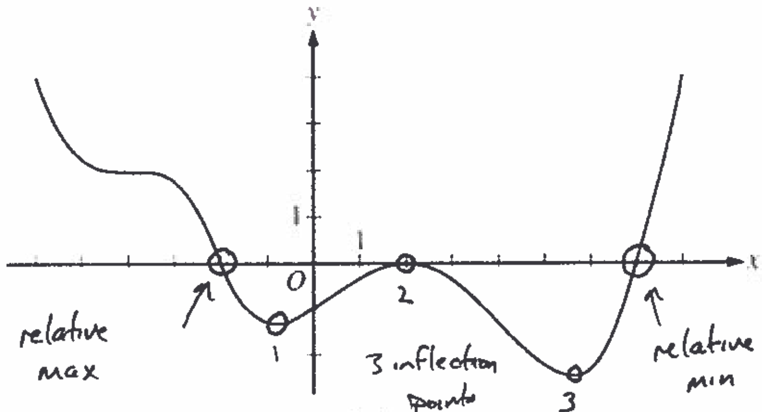
   ~~(A)~~ I only
   ~~(B)~~ I and II only
   (C) I and III only
   ~~(D)~~ II and III only

2. The graph of f', the derivative of a function f, is shown to the right. The domain of f is the closed interval $-2 \le x \le 5$. Which of the following statements is true?



Graph of f'

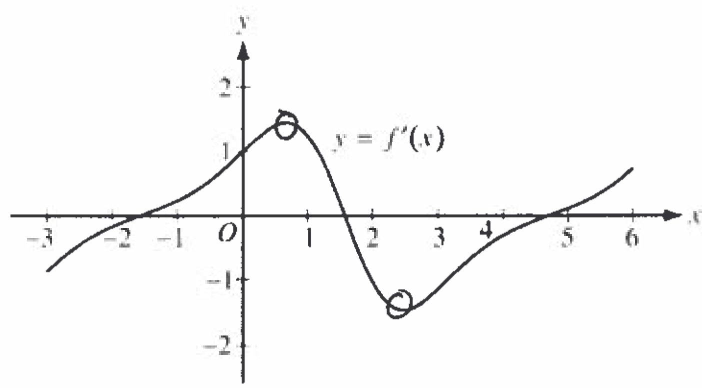
   ~~(A)~~ $f(x)$ is decreasing over $(2,4)$. × f' is decreasing
   ~~(B)~~ $f(x)$ is increasing over $(4, 5)$. × f' is increasing
   (C) $f(x)$ is increasing over $(1, 3)$. ✓
   ~~(D)~~ $f(x)$ has a local maximum at $x = 2$. × f' does
   ~~(E)~~ $f(x)$ has two local extrema on $(-2, 5)$. × one extrema at $x = 3$



Graph of f'

3. The figure above shows the graph of f', the derivative of the function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

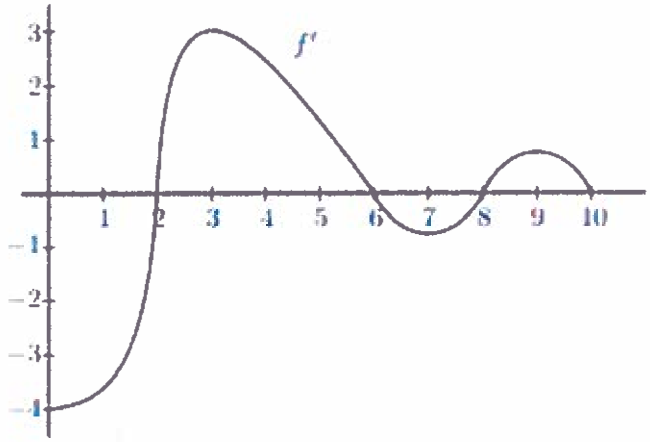
(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection
~~(B)~~ 1 relative minimum, 1 relative maximum, and 4 points of inflection
~~(C)~~ 2 relative minima, 1 relative maximum, and 2 points of inflection
~~(D)~~ 2 relative minima, 1 relative maximum, and 4 points of inflection
~~(E)~~ 2 relative minima, 2 relative maxima, and 3 points of inflection



4. The figure above shows the graph of f', the derivative of the function f, on the interval $[-3, 6]$. If the derivative of the function h is given by $h'(x) = 2f'(x)$, how many points of inflection does the graph of h have on the interval $[-3, 6]$?

f' has 2 extrema
$h'(x)$ has 2 extrema
h has 2 inflection points

~~(A)~~ One
(B) Two
~~(C)~~ Three
~~(D)~~ Four
~~(E)~~ Five



5. The graph above is the graph of the derivative of a function f. Use the graph to answer each of the following questions about f on the interval $(0, 10)$. Justify each answer with 1 sentence.

a) On what interval(s) is f increasing?

$(2,6)$ and $(8,10)$ $f' > 0$ on $(2,6)$ and $(8,10)$

b) On what interval(s) is f decreasing?

$(0,2)$ and $(6,8)$ $f' < 0$ on $(0,2)$ and $(6,8)$

c) On what interval(s) is f concave up?

$(0,3)$ and $(7,9)$ f' is increasing on $(0,3)$ and $(7,9)$

d) On what interval(s) is f concave down?

$(3,7)$ and $(9,10)$ f' is decreasing on $(3,7)$ and $(9,10)$

e) Find the x-coordinates of all relative minima of f.

$x=2$, $x=8$ f' changes from negative to positive at $x=2$ and $x=8$.

f) Find the x-coordinates of all relative maxima of f.

$x=6$ f' changes from positive to negative at $x=6$.

g) Find the x-coordinates of all points of inflection of f.

$x=3$, $x=7$, $x=9$ The graph of f' has extrema, i.e. f' changes from increasing to decreasing or vice versa, at $x=3, 7$, and 9.

h) At what value(s) of x is $f''(x)$ undefined?

$x=2$, f' has a vertical tangent at $x=2$

Fast Workers! Nice Job working through Problem Set A and Problem Set B. Keep the math fresh by working through these spiral problems below.

AP Open-Ended!

- **Each set of spiraled questions this week will be an actual AP open-ended problem from an old exam. Solve with a partner, then check with the scoring guide to see what score you achieved!**

1. Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

   a) On what intervals is f increasing?

   $f'(x) = 3x^2 - 10x + 3$

   $f'(x) = 0 = (3x-1)(x-3)$

   $x = 1/3,\ x = 3$

   f': $+$ on $(-\infty, 1/3)$, $-$ on $(1/3, 3)$, $+$ on $(3, +\infty)$

   f is increasing on $(-\infty, 1/3)$ and $(3, +\infty)$ because $f'(x) > 0$ on $(-\infty, 1/3)$ and $(3, +\infty)$.

   b) On what intervals is the graph of f concave downward?

   $f''(x) = 6x - 10$

   $0 = 2(3x-5)$

   $x = 5/3$

   f'': $-$ on $(-\infty, 5/3)$, $+$ on $(5/3, +\infty)$

   $(-\infty, 5/3)$ f is concave downward on $(-\infty, 5/3)$ because $f''(x) < 0$ and $(-\infty, 5/3)$.

   c) Find the value of k for which f has 11 as its relative minimum.

   Relative Minimum at $x = 3$, f' changes from negative to positive.

   $f(3) = 11$

   $f(3) = 3^3 - 5(3)^2 + 3(3) + k = 11$

   $27 - 45 + 9 + k = 11$

   $-9 + k = 11$

   $\boxed{k = 20}$

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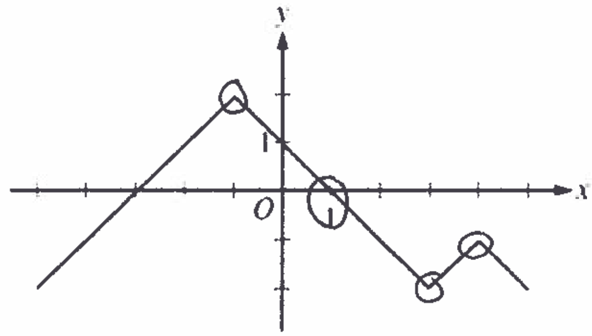
Name ______________ Seat # ______ Date ______________

Pre-AP Calculus Education is Freedom

10.07 – Exit Ticket **Binder Section: ET**



Graph of f'

A. The graph of f', the derivative of f, is shown in the figure above.

1. The function f has a local maximum at $x =$

$x = 1$

~~(A)~~ −3
~~(B)~~ −1
(C) 1
~~(D)~~ 3
~~(E)~~ 4

2. How many points of inflection does the graph of f have?

$x = -1, x = 3, x = 4$

~~(A)~~ 0
~~(B)~~ 1
~~(C)~~ 2
(D) 3
~~(E)~~ 4

3. On what interval(s) is f both increasing and concave down? Justify your answer.

f is both increasing and concave down on $(-1, 1)$ because $f'(x) > 0$ and $f'(x)$ is decreasing on $(-1, 1)$.

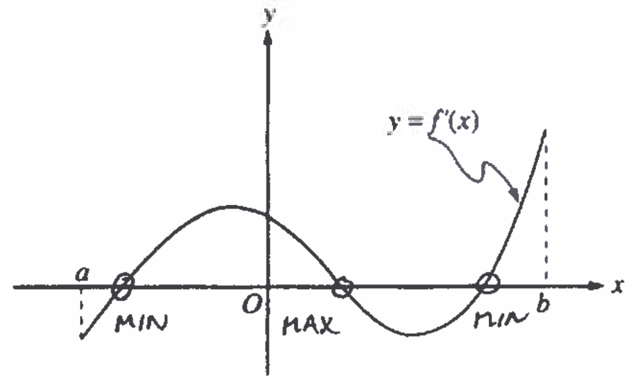
Name ____________________ Seat # ______ Date ______________

Pre-AP Calculus — Education is Freedom

10.07 – Homework — **Binder Section: HW**

Part I: New Material – Analyzing the Graph of $f'(x)$

1. The graph of f', the derivative of f, is shown in the figure to the right. Which of the following describes all relative extrema of f on the open interval (a, b)?



(A) One relative maximum and two relative minima

~~(B)~~ Two relative maxima and one relative minimum

~~(C)~~ Three relative maxima and one relative minimum

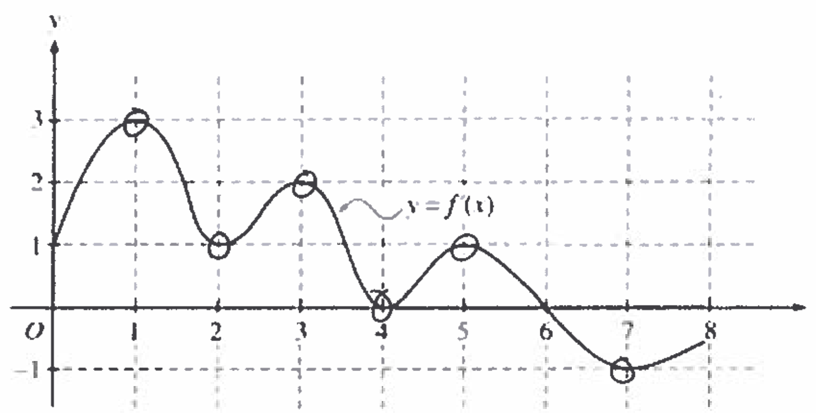
~~(D)~~ One relative maximum and three relative minima

~~(E)~~ Three relative maxima and two relative minima

Justify your answer.

f has two relative minima because f' changes from negative to positive twice, and f' has one relative maximum because f' changes from positive to negative once.

2. The function f is defined on the closed interval $[0, 8]$. The graph of its derivative f' is shown to the right. How many points of inflection does the graph of f have?



~~(A)~~ Two

~~(B)~~ Three

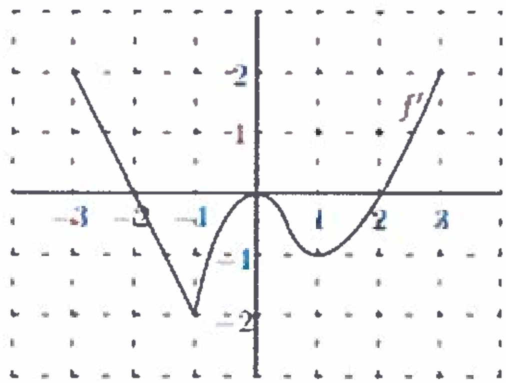
~~(C)~~ Four

~~(D)~~ Five

(E) Six

Justify your answer.

f' changes from increasing to decreasing or decreasing to increasing at x=1, x=2, x=3, x=4, x=5, and x=7, so f has 6 points of inflection.



3. The graph of $f'(x)$, the derivative of $f(x)$, is shown above. The domain of f is the interval $-3 \le x \le 3$. Which of the following is true about the graph of f?

I. f is increasing on $(-3, -2)$. ✓ $f' > 0$

II. f is concave down on $(-3, -1)$. ✓ f' is decreasing

III. The maximum value of $f(x)$ on $(-3, 2)$ is $f(-3)$. x $f(-2) > f(-3)$ since f is increasing on $(-3,-2)$

~~(A)~~ I only
~~(B)~~ II only
~~(C)~~ III only
(D) I and II only
~~(E)~~ II and III only

Part II: Spiral Material – keep the math fresh!

4. Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both decreasing and concave up?

~~(A)~~ $(-\infty, -1)$
~~(B)~~ $\left(-1, \frac{1}{2}\right)$
~~(C)~~ $(-1, 2)$
(D) $\left(\frac{1}{2}, 2\right)$
~~(E)~~ $(2, \infty)$

$f'(x) = 6x^2 - 6x - 12$
$0 = 6(x^2 - x - 2)$
$0 = 6(x-2)(x+1)$
$x = 2, x = -1$

f': $+$ $-$ $+$
$-\infty$ -1 2 $+\infty$

$(-1, 2)$ decreasing

$f''(x) = 12x - 6$
$0 = 6(2x-1)$
$x = 1/2$

f'': $-$ $+$
$-\infty$ $1/2$ $+\infty$

$(1/2, \infty)$ concave up

$\left(\frac{1}{2}, 2\right)$
both decreasing and concave up